Homework Policy

 Assignments will be finalized 1 hour after class ends: do not start the assignment until then

This gives me time to edit the assignment in case some concepts were not covered sufficiently in during class

• The Syllabus has been updated to display this info.

Describing data with charts: histograms

General quantitative data: A histogram is a continuous barplot for ranges of a variable.

TABLE 1.1

IQ test scores for 60 randomly chosen fifth-grade students

145	139	126	122	125	130	96	110	118	118
101	142	134	124	112	109	134	113	81	113
123	94	100	136	109	131	117	110	127	124
106	124	115	133	116	102	127	117	109	137
117	90	103	114	139	101	122	105	97	89
102	108	110	128	114	112	114	102	82	101

Class	Count
80 - 89	3
90 – 99	4
100 - 109	14
110 - 119	17
120 – 129	11
130 - 139	9
140 - 149	2



Things to notice:

Bar graphs separate by *value*; histograms separate by *range*.



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- Bar graphs have spaces between columns; histograms do not



Things to notice:

- Bar graphs separate by *value*; histograms separate by *range*.
- Bar graphs have spaces between columns; histograms do not
- Both have frequency versions



Things to notice:

- Bar graphs separate by *value*; histograms separate by *range*.
- Bar graphs have spaces between columns; histograms do not
- Both have frequency versions and both have proportion versions



Histograms: distribution shape

Terminology: Skewed data vs. symmetric data * Skew is in the direction of the "longer" side



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Describing densities (Section 1.4)

- For symmetric densities, mean and median are the same
- For skewed densities, mean is pulled in direction of skew
- Example: Median below is 66K; mean is much higher



Describing data with densities (Section 1.4)

Density (roughly): a curve which describes data and where it falls

We can find a density that well-approximates a histogram:



Note: this is a density histogram; area under bars is $1 \in \mathbb{R}$ and $1 \in \mathbb{R}^{3/2}$

Density definition (Section 1.4)

Density (exactly): a positive line that has area exactly area 1 between it and the horizontal axis.

For any two numbers, we can find the area under a density between them. It will always be less than or equal to 1.



Describing data with densities (Section 1.4)

Use/purpose of a density:

- Consider: every histogram represents a sample from a larger population
- A density is like our **best guess** at the true distribution of the population, given the sample
- For any 2 numbers, area under the density between them is our best guess at the true % between them in the population



Densities have many properties of histograms:

- Median is the point with 50% of the area to the left (and right)
- *p*-th percentile is the point with *p*% of area to left
 - * Q1 is 25th percentile; Q3 is 75th percentile
- Mode is the highest point of the curve (may not be unique)

- Mean is the center of mass (balance point)
- Right/left skew are analgous

Describing densities (Section 1.4)

For symmetric densities, mean and median are the sameFor skewed densities, mean is pulled in direction of skew



Simple densities

- Densities don't have to be curvy.
- Both of these are densities because the area underneath is 1.
- Left side: **uniform** density. All equal-length intervals take up the same proportion of the population.



In-class exercise

What is the median of this density? mean? Q1?



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Answer: (15, 15, 12.5)

Describing data with densities (Section 1.4)

- Many different "reasonable" densities
- Not all are mathematically convenient
- Sometimes, worse fitting density is chosen for convenience.
- Left: we fit the "best-fitting" density to the histogram
- Right: we fit the Normal density:





- Symmetric, unimodal, and bell-shaped
- Center and spread are controlled by two *parameters*:

 μ the mean, and σ the standard deviation

Parameters are like mean and standard error of real data.
 σ extends to "inflection point" of curve



Center and spread are controlled by:

 μ the mean, and σ the standard deviation

When you change μ and σ , you change the density:



To fit a Normal density to data, set μ and σ to the sample mean and standard error.

$$\bar{x} = 115.0, \ s = 14.8$$
 $\bar{x} = 121.9, \ s = 11.9$



Why use the Normal density?

- Normal densities look like many chance outcomes (e.g. coin flip counts)
- ... therefore, many real data sets are closely Normal
- Convenience: many stat methods work well w/Normal

Convenience2: has handy properties to describe data (next)
 But be careful! Some data sets are obviously non-Normal.
 Important to recognize when this occurs (later in course).

Describing data with the Normal (Section 1.4)

68-95-99.7 Rule: under a Normal density with mean μ and standard deviation σ , there is:

- 68% of the data within 1σ of μ
- 95% of the data within 2σ of μ
- 99.7% of the data with 3σ of μ



Describing data with the Normal (Section 1.4)

Example of 68-95-99.7. Suppose heights of Hobbits follow a Normal density with $\mu = 30$ inches and $\sigma = 1.5$ inches. Then:

- 68% of Hobbits are within 1.5 inches of 30 inches
- 95% of Hobbits are within 3 inches of 30 inches
- 99.7% of Hobbits are within 4.5 inches of 30 inches



Hobbits

Rule: (68, 95, 99.7)% of data is within $(1, 2, 3)\sigma$ of μ

Heights of Hobbits are $\mathcal{N}(30, 1.5)$. Suppose Frodo is 33 inches tall. What proportion of Hobbits are shorter than Frodo?





Rule: (68,95,99.7)% of data is within $(1,2,3)\sigma$ of μ

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Answer: 97.5% = + = - 23/32

Rule: (68, 95, 99.7)% of data is within $(1, 2, 3)\sigma$ of μ

Heights of Hobbits are $\mathcal{N}(30, 1.5)$. Suppose Frodo is 33 inches tall. What proportion of Hobbits are shorter than Frodo?



Heights of Elves are $\mathcal{N}(72,3)$. Suppose Legolas is 78 inches tall (6-foot-6!). What proportion of Elves are shorter than Legolas?



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Answer: 97.5%

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Heights of Hobbits are $\mathcal{N}(30, 1.5)$. Suppose Frodo is 33 inches tall. What proportion of Hobbits are shorter than Frodo?



Heights of Elves are $\mathcal{N}(72, 3)$. Suppose Legolas is 78 inches tall (6-foot-6!). What proportion of Elves are shorter than Legolas?



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Hobbits are $\mathcal{N}(30, 1.5)$:

Elves are $\mathcal{N}(72,3)$:





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Hobbits are $\mathcal{N}(30, 1.5)$:

Elves are $\mathcal{N}(72,3)$:



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For a point x from a $\mathcal{N}(\mu, \sigma)$ population, the z-score is defined

$$z = \frac{x - \mu}{\sigma}$$

Data at same percentile have the same z-score (& vice-versa)

Hobbits are $\mathcal{N}(30, 1.5)$, Frodo is 33 inches tall. His *z*-score is

Elves are $\mathcal{N}(72, 3)$, Legolas is 78 inches tall. His *z*-score is

$$\frac{33-30}{1.5} = 2 \qquad \qquad \frac{78-72}{3} = 2$$

*** So a z-score **counts sigmas** between x and μ !

$$z = \frac{x - \mu}{\sigma}$$

- **z** values are have a Normal $\mu = 0$, $\sigma = 1$ density (called "Standard Normal")
- ... thus, the 68 95 99.7 rule applies to z-scores too
- Notice z = 2 is just 2σ when $\sigma = 1$
- ... and 97.5% of z-values are below z = 2



$$z = \frac{x-\mu}{\sigma}$$

Frodo's *z*-score:
$$\frac{33 - 30}{1.5} = 2$$

Legolas's z-score:
$$\frac{78-72}{3} = 2$$



What happens when things aren't as easy?

Frodo's z-score:
$$\frac{31-30}{1.5} \approx 0.66$$
 Legolas's z-score: $\frac{77-72}{3} \approx 1.66$





- Frodo and Legolas are now different percentiles of their populations
- How do we know what percentiles they are?
- Can't use 68-95-99.7 rule: their z-scores aren't integers

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Normal tables (Section 1.4)

- Every z-score has a cumulative proportion before it, given by the Standard Normal density
- z proportions cannot be computed directly
- Need to use a table (Table A in your textbook):

0.0 0.5000 0.5040 0.5080 0.5160 0.199 0.5239 0.1 0.5398 0.5438 0.5478 0.5517 0.5557 0.5567 0.5636 0.2 0.5793 0.5832 0.5871 0.5557 0.5557 0.5636 0.3 0.5120 0.5812 0.5910 0.5548 0.6026 0.4 .6554 0.6526 0.6628 0.6664 0.6700 67120 0.6772 0.5 0.6915 0.6950 0.6985 0.7019 0.7054 0.7788 0.7123 0.6772 0.5 0.6915 0.6985 0.7019 0.77389 0.7422 0.7454 0.6 0.7257 0.7291 0.7324 0.7738 0.7734 0.7734 0.8 0.7581 0.7910 0.7939 0.7857 0.8023 0.8051 0.9 0.8159 0.8186 0.8212 0.8238 0.8244 0.8239 0.8315 1.0 0.8413 0.8460 0.8445	z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
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1.6 0.9452 0.9463 0.9474 0.9484 0.9495 0.9505 0.9515	1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406
	1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515

- z-scores in the margins
- Proportions in the table
- Top margin is "completion" of side margin

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Normal tables (Section 1.4)

- z-scores can also be negative
- If a Hobbit is $26\frac{1}{4}$ inches, the *z*-score is $\frac{26.25-30}{1.5} = \frac{-3.75}{1.5} = -2.5$. What % of Hobbits are shorter?

z	.00	.01	.02	.03	.04
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618

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