

Suppose a large population of people are divided accordingly:

Boston Red Sox Fans	30%
NY Yankees Fans	30%
Houston Astros Fans	15 %
Oakland A's Fans	10%
SF Giants Fans	15 %

In an experiment you sample two people independently and at random.

**Question 1:** What is the probability that both people will be fans of teams that are either from Texas or California?

- Solution: Define the events

$A$  =The first person is a fan of a T/C team

$B$  =The second person is a fan of a T/C team

This means  $A = \{\text{first person is Houston fan } \textit{or} \text{ Oakland fan } \textit{or} \text{ SF fan}\}$  and the same holds true for  $B$  but for the second person. Since the probability of an event is the sum of the probability of its possible outcomes,

$$P(A) = P(\text{Houston fan}) + P(\text{Oakland fan}) + P(\text{SF fan}) = .15 + .10 + .15 = .40$$

and the same is true for  $P(B)$ . Since the problem says the samples are independent,

$$P(A \text{ and } B) = P(A)P(B) = 0.40^2 = 0.16$$

- Alternate (longer but more basic) solution: denote each outcome of a *single* sample by the first letter of the city. Then our sample space is

$$S = \{BB, BN, BH, BO, BS, \\ NB, NN, NH, NO, NS, \\ HB, HN, HH, HO, HS, \\ OB, ON, OH, OO, OS, \\ SB, SN, SH, SO, SS\}$$

Since the two samples are independent, we can calculate the probabilities of each of these outcomes using the multiplication rule and the population proportions. Then  $P(\text{both people will be fans of a T or C team})$  is just the sum of the probabilities of the outcomes above that have two Texan or Californian teams. You can check yourself that these add to 0.16.

**Question 2:** What is the probability that at least one person will be a T/C fan?

- Solution: Note that the event “at least one person will be a T/C fan” is the *complement* of the event “both people are **not** T/C fans”. Therefore by the complement rule

$$P(\text{at least one person will be a T/C fan}) = 1 - P(\text{both people are **not** T/C fans})$$

So this reduces the problem to solving for  $P(\text{both people are **not** T/C fans})$ . Using events  $A$  and  $B$  from before, note that

$$\{\text{both people are **not** T/C fans}\} = \{A^C \text{ and } B^C\}$$

Again by the complement rule, we have  $P(A^C) = 1 - P(A) = 0.60$  and the same holds for  $B^C$ . Thus by independence

$$P(\text{both people are **not** T/C fans}) = P(A^C \text{ and } B^C) = P(A^C)P(B^C) = 0.60^2 = 0.36$$

So, the answer is  $1 - 0.36 = 0.64$ .

- Alternate (longer but more basic) solution: using the sample space  $S$  as before, just sum the probabilities of the outcomes that bring about the event “at least one person will be a fan of a T/C team”. This method is conceptually easier but takes longer, and there is more room for calculator error.