Suppose a large population of people are divided accordingly:

Boston Red Sox Fans	30%
NY Yankees Fans	30%
Houston Astros Fans	15 %
Oakland A's Fans	10%
SF Giants Fans	15 %

In an experiment you sample two people independently and at random.

Question 1: What is the probability that both people will be fans of teams that are either from Texas or California?

• Solution: Define the events

A =The first person is a fan of a T/C team B =The second person is a fan of a T/C team

This means $A = \{$ first person is Houston fan *or* Oakland fan *or* SF fan $\}$ and the same holds true for *B* but for the second person. Since the probability of an event is the sum of the probability of its possible outcomes,

P(A) = P(Houston fan) + P(Oakland fan) + P(SF fan) = .15 + .10 + .15 = .40

and the same is true for P(B). Since the problem says the samples are independent,

$$P(A \text{ and } B) = P(A)P(B) = 0.40^2 = 0.16$$

• Alternate (longer but more basic) solution: denote each outcome of a *single* sample by the first letter of the city. Then our sample space is

$$S = \{BB, BN, BH, BO, BS, \\NB, NN, NH, NO, NS, \\HB, HN, HH, HO, HS, \\OB, ON, OH, OO, OS, \\SB, SN, SH, SO, SS\}$$

Since the two samples are independent, we can calculate the probabilities of each of these outcomes using the multiplication rule and the population proportions. Then P(both people will be fans of a T or C team) is just the sum of the probabilities of the outcomes above that have two Texan or Californian teams. You can check yourself that these add to 0.16.

Question 2: What is the probability that at least one person will be a fan of a T/C team?

• Solution: Note that the event "at least one person will be a T/C fan" is the *complement* of the event "both people are **not** T/C fans". Therefore by the complement rule

 $P(\text{at least one person will be a T/C fan}) = 1 - P(\text{both people are$ **not** $T/C fans})$

So this reduces the problem to solving for P(both people are not T/C fans). Using events A and B from before, note that

{both people are **not** T/C fans} = { A^C and B^C }

Again by the complement rule, we have $P(A^C) = 1 - P(A) = 0.60$ and the same holds for B^C . Thus by independence

 $P(\text{both people are$ **not** $T/C fans}) = P(A^C \text{ and } B^C) = P(A^C)P(B^C) = 0.60^2 = 0.36$

So, the answer is 1 - 0.36 = 0.64.

• Alternate (longer but more basic) solution: using the sample space S as before, just sum the probabilities of the outcomes that bring about the event "at least one person will be a fan of a T/C team". This method is conceptually easier but takes longer, and there is more room for calculator error.